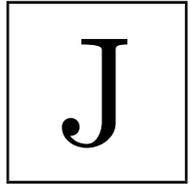


# KANGAROO 2013



**Junior**  
9–10 grades

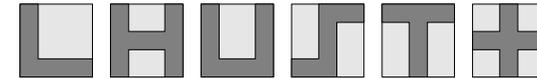
*Time allowed: 75 min*  
*Calculators are not permitted*

## Questions for 3 points

24. We call “changesum” the procedure to make from a list of three numbers the new list by replacing each number by the sum of the other two. For example, from  $\{3, 4, 6\}$  “changesum” gives  $\{10, 9, 7\}$  and a new “changesum” leads to  $\{16, 17, 19\}$ . If we begin with the list  $\{1, 2, 3\}$ , how many consecutive “changesums” will be required to get the number 2013 in the list?  
A) 8 B) 9 C) 10 D) 2013 E) 2013 will never appear
25. The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 are written around the circle in arbitrary order. By adding to each number it’s two neighbours, we obtain 10 sums. What is the maximum possible value of the smallest of these sums?  
A) 14 B) 15 C) 16 D) 17 E) 18
26. On 22 cards positive integers from 1 to 22 are written. With these cards 11 fractions have been made. What is the greatest number of these fractions that can have integer values?  
A) 7 B) 8 C) 9 D) 10 E) 11
27. How many triangles are there, whose vertices are chosen from the vertices of a given regular polygon with 13 sides, and such that the centre of the circumcircle of the polygon is inside of the triangle?  
A) 72 B) 85 C) 91 D) 100 E) Other value
28. A car left point  $A$  and drove along the straight road at a speed of 50 km/h. Then every hour a car left point  $A$ , and each next car was 1 km/h faster than the previous one. The last car (at a speed of 100 km/h) left 50 hours after the first one. What is the speed of the car which was in front of all the column 100 hours later after the start of the first car?  
A) 50 km/h B) 66 km/h C) 75 km/h D) 84 km/h E) 100 km/h
29. 100 trees (oaks and birches) grow along a road. The number of trees between any two oaks does not equal 5. What greatest number of oaks can be among these 100 trees?  
A) 48 B) 50 C) 52 D) 60 E) The situation is not possible
30. Yurko was walking down the street when he saw a tractor that was pulling a long pipe. To measure its length, Yurko walked along the pipe against the movement of the tractor and counted 20 steps. Then he walked along the pipe with the movement of the tractor and counted 140 steps. Yurko’s step equals 1 m. His and tractor’s speed were constant. What is the length of the pipe?  
A) 30 m B) 35 m C) 40 m D) 48 m E) 80 m

1. The number 200013 – 2013 is not divisible by:  
A) 2 B) 3 C) 5 D) 7 E) 11

2. Mary shades various shapes on square sheets of paper, as shown below.

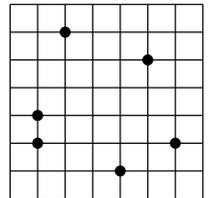


How many of these figures have perimeter equal to the perimeter of the sheet of paper?  
A) 2 B) 3 C) 4 D) 5 E) 6

3. Mrs. Margareth bought 4 cobs of corn for everyone in her 4-member family. In the shop she got the discount the shop offered: “Corn sale! 1 cob for 20 cents! Every sixth cob is for free!”. How much did she pay?  
A) 0,80 EUR B) 1,20 EUR C) 2,80 EUR D) 3,20 EUR E) 80 EUR

4. Three of the numbers 2, 4, 16, 25, 50, 125 have product 1000. What is their sum?  
A) 70 B) 77 C) 131 D) 143 E) None of the previous

5. Six points are marked on a square grid with cell of size 1. What is the smallest area of a triangle with vertices at marked points?  
A)  $\frac{1}{4}$  B)  $\frac{1}{3}$  C)  $\frac{1}{2}$  D) 1 E) 2



6. Adding  $4^{15}$  to  $8^{10}$ , Mihai has obtained a number which is a power of 2. The number equals:  
A)  $2^{10}$  B)  $2^{15}$  C)  $2^{20}$  D)  $2^{30}$  E)  $2^{31}$

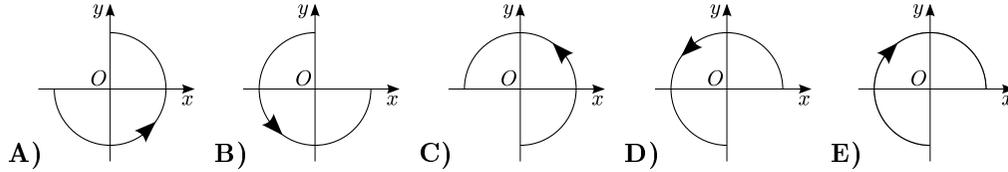
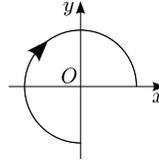
7. On the outside a cube is painted with black and white squares as if it was built of four white and four black smaller cubes. Which of the following is a correct building scheme for this cube?



- A) B) C) D) E)

8. The number  $n$  is the largest positive integer for which  $4n$  is a 3-digit number, and  $m$  is the smallest positive integer for which  $4m$  is a 3-digit number. What is the value of  $4n - 4m$ ?
- A) 900   B) 899   C) 896   D) 225   E) 224

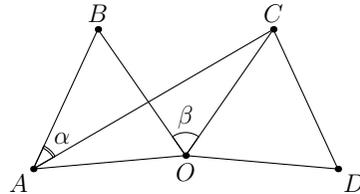
9. Consider a three-quarter circle with center  $O$  and an orientation arrow as indicated in the picture on the right. What is the position of the oriented three-quarter circle when it is first rotated counterclockwise by  $90^\circ$  around  $O$  and then reflected at the  $x$ -axis?



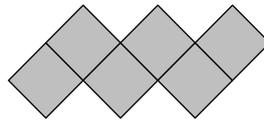
10. Which of the following numbers is the largest?
- A)  $\sqrt{20} \cdot \sqrt{13}$    B)  $\sqrt{20} \cdot 13$    C)  $20 \cdot \sqrt{13}$    D)  $\sqrt{201} \cdot 3$    E)  $\sqrt{2013}$

**Questions for 4 points**

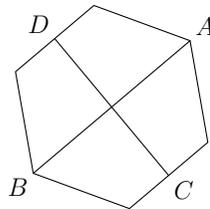
11. Triangle  $COD$  is the image of the equilateral triangle  $AOB$  upon rotation around  $O$ , whereby  $\beta = \angle BOC = 70^\circ$ . Determine the angle  $\alpha = \angle BAC$ .
- A)  $20^\circ$    B)  $25^\circ$    C)  $30^\circ$    D)  $35^\circ$    E)  $40^\circ$



12. The figure below shows zigzag of six unit squares. Its perimeter is 14. What is the perimeter of a zigzag made in the same way consisting of 2013 squares?
- A) 2022   B) 4028   C) 4032   D) 6038   E) 8050

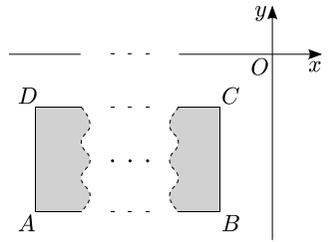


13. The segment  $AB$  connects two opposite vertices of a regular hexagon. The segment  $CD$  connects the midpoints of two opposite sides. Find the product of the lengths of  $AB$  and  $CD$  if the area of the hexagon is 60.
- A) 40   B) 50   C) 60   D) 80   E) 100



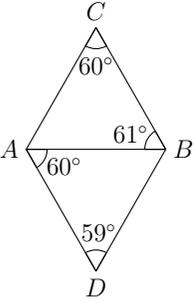
14. A class of students had a test. If each boy had got 3 points more for the test, then the average result of the class would had been 1,2 points higher than now. How many percent of the students of the class are girls?
- A) 20%   B) 30%   C) 40%   D) 60%   E) It is impossible to determine

15. The sides of rectangle  $ABCD$  are parallel to the coordinate-axes (see pic.). We calculate for each of these points the number  $y$ -coordinate  $\div$   $x$ -coordinate. Which of the four points gives the smallest number?
- A)  $A$    B)  $B$    C)  $C$    D)  $D$    E) It is impossible to determine



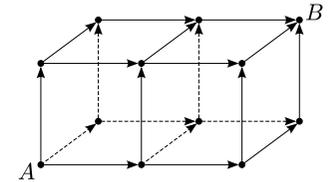
16. Today John and his son are celebrating their birthday. John multiplied correctly his age by the age of his son and obtained the answer 2013. In which year was John born?
- A) 1981   B) 1982   C) 1953   D) 1952   E) More information is needed

17. John wanted to draw two equilateral triangles attached to get a rhombus. But he did not hit correctly all the distances and, once he had done, Jane measured the four angles and saw that they were not equal (see pic.). Which of the five segments of the figure is the longest?
- A)  $AD$    B)  $AC$    C)  $AB$    D)  $BC$    E)  $BD$



18. Five consecutive positive integers have the following property: three of them have the same sum as the sum of other two. How many such sets of integers exist?
- A) 0   B) 1   C) 2   D) 3   E) More than 3

19. What is the number of all different paths going from the point  $A$  to the point  $B$  at the given graph?
- A) 6   B) 8   C) 9   D) 12   E) 15



20. A six-digit positive integer is given. The sum of its digits is an even number, the product of its digits is an odd number. Which statement about this number might be correct?
- A) Either two or four digits of the number are even   B) Such a number cannot exist  
C) The amount of the odd digits of the number is odd   D) The number has six different digits   E) None of the above

**Questions for 5 points**

21. How many decimal places are there in the decimal number  $\frac{1}{1024000}$ ?
- A) 10   B) 12   C) 13   D) 14   E) 1024000

22. How many positive integers are multiples of 2013 and have exactly 2013 positive divisors (including 1 and the number itself)?
- A) 0   B) 1   C) 3   D) 6   E) Another number

23. The picture shows a polygon divided into five isosceles triangles with top angles  $24^\circ$ ,  $48^\circ$ ,  $72^\circ$ ,  $96^\circ$  and  $120^\circ$  – the first multiples of the smallest top angle. All top angles have an integer number of degrees. We want to make a similar picture with as many non-overlapping triangles as possible. How many degrees is the smallest top angle in that case?
- A) 1   B) 2   C) 3   D) 6   E) 8

