KANGAROO 2022

S

EN

Student
11–12 grades

Questions for 3 points

**1.** Carola is forming the four-digit number 2022 using some matches from a box. The box originally contained 30 matches. She has already started and formed the first two digits, as shown in the diagram. How many matches will remain in the box when she has finished forming 2022?
**A)** 20   **B)** 19   **C)** 10   **D)** 9   **E)** 5
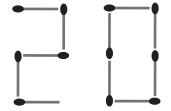
2. How many positive three-digit integers are divisible by 8?
A) 111   **B)** 112   **C)** 113   **D)** 124   **E)** 125

3. Bella is older than Charlie and younger than Lily. Teddy is older than Bella. Which two people could be the same age?
A) Charlie and Teddy   **B)** Teddy and Lily   **C)** Lily and Charlie   **D)** Bella and Lily   **E)** Teddy and Bella

4. The product of the digits of a 10-digit integer is 15. What is the sum of the digits of this integer?
A) 8   **B)** 12   **C)** 15   **D)** 16   **E)** 20

5. Four circles, each of radius 1, intersect as shown. What is the perimeter of the shaded region?
A) Greater than $\frac{3\pi}{2}$ and less than 2π **B)** $\frac{3\pi}{2}$ **C)** 2π **D)** π **E)** π^2
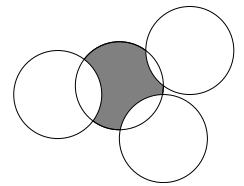
6. On Nadya's smartphone, the diagram shows how much time she spent last week on each of her apps. This week she chose two of these apps and halved the time spent on each of them, but the amount of time spent on each of the other two apps remained the same. Which of the following **cannot** be the diagram for this week?
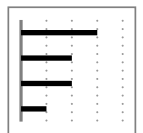
A)   **B)**   **C)**   **D)**   **E)**

7. How many real solutions does the equation $(x-2)^2 + (x+2)^2 = 0$ have?
A) 0   **B)** 1   **C)** 2   **D)** 3   **E)** 4

8. Four lines intersect forming eight equal angles. Which black arc has the same length as the smallest circle?
A) A **B)** B **C)** C **D)** D **E)** E
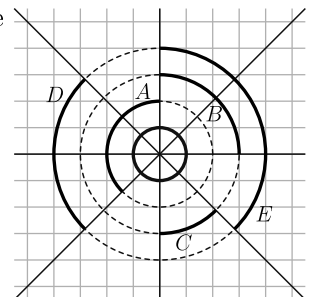
9. Let a, b, c be non-zero numbers. The numbers $-2a^4b^3c^2$ and $3a^3b^5c^{-4}$ have the same sign. Which of the following is definitely true?
A) $ab > 0$ **B)** $b < 0$ **C)** $c > 0$ **D)** $bc > 0$ **E)** $a < 0$

10. Mike has marked the points A, B, C and D in this order on a straight line, as shown in the diagram. What is the distance between the midpoint of AB and the midpoint of CD if $AC = 12$ and $BD = 18$?

A) 15   **B)** 12   **C)** 18   **D)** 6   **E)** 9

Questions for 4 points

11. When he looks at the water meter in his bathroom, Tony notices that all the digits on the meter are different: 91.876 m^3. How much water will be used until the next time all the digits on the meter are different?
A) 0.006 m^3 B) 0.034 m^3 C) 0.086 m^3 D) 0.137 m^3 E) 1.048 m^3

12. A large square is divided into two squares and two rectangles, as shown. The vertices of the shaded quadrilateral are the midpoints of the sides of the two squares. The area of the shaded quadrilateral is 3. What is the area of the unshaded part of the large square?
A) 12   B) 15   C) 18   D) 21   E) 24
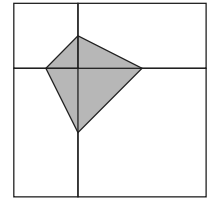
13. What is the greatest common divisor of $2^{2021} + 2^{2022}$ and $3^{2021} + 3^{2022}$?
A) 2^{2021} B) 1   C) 2   D) 6   E) 12

14. The map shows a region with 16 cities connected by roads. The government wants to build electricity power plants in some of the cities. Each power plant can provide enough electricity for the city where it is sited and any cities connected to that city by a single road. What is the smallest number of power plants that need to be built?
A) 3   B) 4   C) 5   D) 6   E) 7
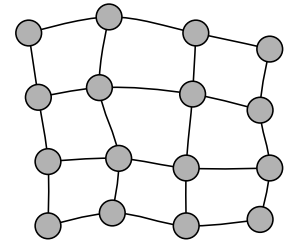
15. Three children asked their grandmother how old she was. She replied by asking them to guess her age. One child said she was 75, one said she was 78 and one said she was 81. It turned out that one of the guesses was wrong by 1 year, one was wrong by 2 years and one was wrong by 4 years. What is the grandmother's age?
A) 76   B) 77   C) 79   D) 80   E) Cannot be determined exactly

16. David writes, in increasing order, all the integers from 2 to 2022 which use only 0s and 2s. What is the number in the middle of his list?
A) 200   B) 220   C) 222   D) 2000   E) 2002

17. A cuboid of surface area S is cut by six planes as shown. Each plane is parallel to a face. Now the cuboid is separated in 27 smaller parts. What is the total surface area of all 27 smaller parts?
A) $2\,S$ B) $\dfrac{5}{2}\,S$ C) $3\,S$ D) $4\,S$ E) None of the previous
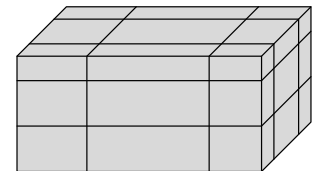
18. Five numbers form an increasing sequence. They have a mean of 24. The mean of the first three numbers is 19 and the mean of the last three numbers is 28. What is the third number?
A) 20   B) 21   C) 22   D) 23   E) 24

19. Which of the pairs of pieces below can be put together to build the shape shown in the diagram?
A)   B)   C)   D)
E)
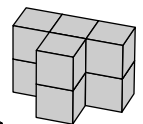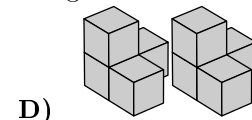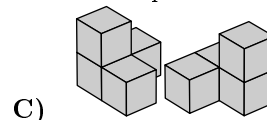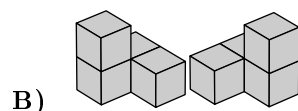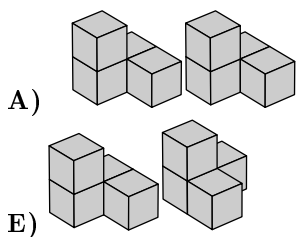
20. A rectangle is divided into 11 smaller rectangles, as shown in the diagram. All 11 rectangles are similar to the original large rectangle. The orientation of the 9 smallest rectangles is the same as the largest. The length of the base of the smallest rectangle is 1. What is the perimeter of the large rectangle?
A) 20   B) 24   C) 27   D) 30   E) 36
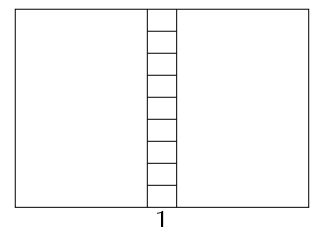
Questions for 5 points

21. A circle with centre $(0;0)$ has radius 5. At how many points on the perimeter of the circle are both coordinates integers?
**A)** 5   **B)** 8   **C)** 12   **D)** 16   **E)** 20
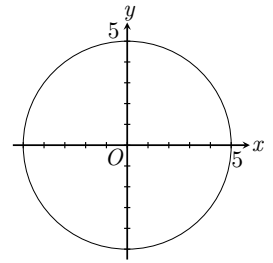
**22.** How many positive three-digit integers are there that are equal to five times the product of their digits?
**A)** 1   **B)** 2   **C)** 3   **D)** 4   **E)** 5

23. The numbers 1 to 10 are placed, once each, in the circles of the figure shown. The sum of the numbers in the top row is 24; the sum of the numbers in the bottom row is also 24 and the sum of the numbers in the right column is 25. What number is in the circle containing the question mark?
A) 2   **B)** 4   **C)** 5   **D)** 6   **E)** None of the previous
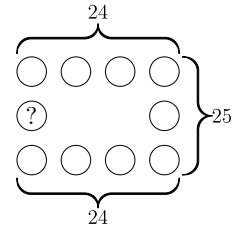
24. The vertices of a square are $(1;1)$, $(2;1)$, $(2;2)$ and $(1;2)$. Each point $(x;y)$ on the square is moved to $\left(\dfrac{1}{x};\dfrac{1}{y}\right)$. What will the resulting figure look like?

A)   **B)**   **C)**   **D)**   **E)**
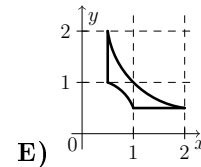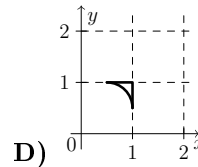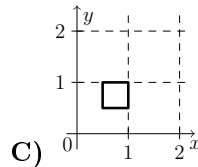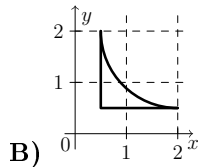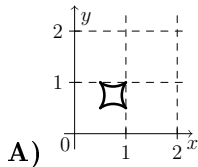
25. The vertices of a 20-gon are numbered from 1 to 20 in such a way that the numbers of adjacent vertices differ by either 1 or 2. The sides of the 20-gon whose ends differ by only 1 are colored red. How many red sides are there?
A) 1   **B)** 2   **C)** 5   **D)** 10   **E)** There are multiple possibilities

26. Two rectangles are inscribed inside a triangle ABC. The dimensions of the rectangles are 1×5 and 2×3, as shown. What is the height of the triangle with base BC?
A) 3   **B)** $\dfrac{7}{2}$ **C)** $\dfrac{8}{3}$ **D)** $\dfrac{16}{5}$ **E)** None of the previous
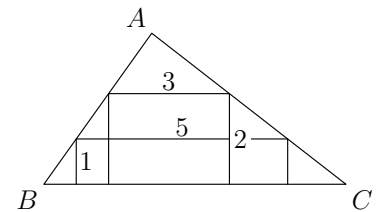
27. Let N be a positive integer greater than 2022. How many integers are there between $\sqrt{N^2+N+1}$ and $\sqrt{9N^2+N+1}$?
A) $N+1$ **B)** $2N-1$ **C)** $2N$ **D)** $2N+1$ **E)** $3N$

28. A regular hexagonal prism has its top corners shaved off, as shown. The top face becomes a smaller regular hexagon and the 6 rectangular faces around the middle become 12 isosceles triangles of two different sizes. What fraction of the volume of the original prism has been lost?

A) $\dfrac{1}{12}$ **B)** $\dfrac{1}{6}$ **C)** $\dfrac{1}{4\sqrt{3}}$ **D)** $\dfrac{1}{6\sqrt{2}}$ **E)** $\dfrac{1}{6\sqrt{3}}$
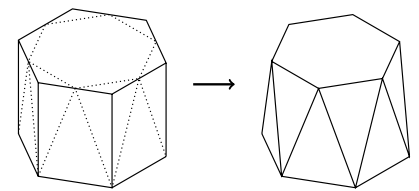
29. A grocer has twelve different integer weights from 1 kg to 12 kg. She splits them into three groups of four weights each. The total weight of the first group is 41 kg and of the second is 26 kg. Which of the following weights is in the same group as the weight of 9 kg?
A) 3 kg   **B)** 5 kg   **C)** 7 kg   **D)** 8 kg   **E)** 10 kg

30. Two circles cut a rectangle $AFMG$, as shown. The line segments outside the circles have length $AB = 8$, $CD = 26$, $EF = 22$, $GH = 12$ and $JK = 24$. What is the length of LM?
A) 15   **B)** 17   **C)** 14   **D)** 16   **E)** 18