24. A square has two of its vertices on a semicircle and the other two on the diameter of the semicircle as shown. The length of the diameter is 2. What is the area of the square?
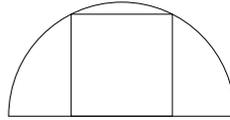
A) $\dfrac{4}{5}$ **B)** $\dfrac{\pi}{4}$ **C)** 1   **D)** $\dfrac{4}{3}$ **E)** $\dfrac{2}{\sqrt{3}}$

25. Two dots are marked in a disc that is rotating around its centre. One of them is 3 cm further than the other to the centre of the disc and moves at a constant speed that is 2.5 times as fast as the other. What is the distance from the centre of the disc to this far point?
A) 10 cm   **B)** 9 cm   **C)** 8 cm   **D)** 6 cm   **E)** None of the previous

26. How many planes pass through exactly three vertices of a given cube?
A) 0   **B)** 2   **C)** 4   **D)** 8   **E)** 12

27. A graph consists of 16 vertices and some edges that connect them, as in the picture. An ant is now at the vertex labelled A. At each move, it can walk from one vertex to any neighbouring vertex crawling along a connecting edge. At which of the vertices labelled P, Q, R, S, T can the ant be after 2019 moves?
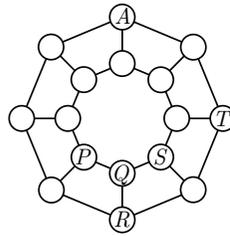

A) Only P, R or S, not Q and T **B)** Only P, R, S or T, not Q
C) Only Q **D)** Only T **E)** All of these vertices are possible

28. What is the least number of elements we have to delete from the set $\{10, 20, 30, 40, 50, 60, 70, 80, 90\}$ so that the product of the elements remaining in the set is a perfect square?
A) 1   **B)** 2   **C)** 3   **D)** 4   **E)** 5

29. Given triangle ABC of area S, let D be the midpoint of BC. Take points P, Q, R on lines AB, AD, AC, respectively, as shown in the picture, and such that $AP = 2 \cdot AB$, $AQ = 3 \cdot AD$ and $AR = 4 \cdot AC$. What is the area of triangle PQR?
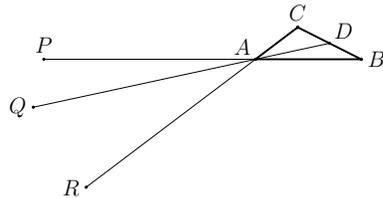
A) S **B)** $2S$ **C)** $3S$ **D)** $\frac{1}{2}S$ **E)** 0 (i.e. P, Q, R are collinear)

30. If any digit of a given 4-digit number is deleted, the resulting 3-digit number is a divisor of the original number. How many 4-digit numbers have this property?
A) 5   **B)** 9   **C)** 14   **D)** 19   **E)** 23

 KANGAROO 2019   **J**

Junior
9–10 grades

Questions for 3 points

1. $20 \cdot 19 + 20 + 19 =$
A) 389   **B)** 399   **C)** 409   **D)** 419   **E)** 429

2. A model train takes exactly 1 minute and 11 seconds for each round on a course. How long does it take for six rounds?
A) 6 minutes 56 seconds   **B)** 7 minutes 6 seconds   **C)** 7 minutes 16 seconds
D) 7 minutes 26 seconds   **E)** 7 minutes 36 seconds

3. A barber wants to write the word **DAILU** on a board in such a way that a client looking in to the mirror reads the word correctly. How should the barber write it on the board?
A) DAILU   **B) ꓷAILU**   **C) UꓘIAꓷ**   **D) ULIAꓷ**   **E) ꓷAILU**

4. The numbers 1, 2, 3 and 4 are each written in different cells of the 2×2 table. After that, the sum of the numbers in each row and column is calculated. Two of these sums are 4 and 5. What are the other two sums?
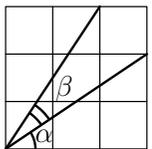

A) 6 and 6   **B)** 3 and 5   **C)** 4 and 5   **D)** 4 and 6   **E)** 5 and 6

5. A park has five gates. Monica wants to enter through one gate and to exit through a different one. In how many ways can she enter and exit the park?
A) 10   **B)** 15   **C)** 16   **D)** 20   **E)** 25

6. Which of the following statements is true for the marked angles in the given figure of nine identical squares?
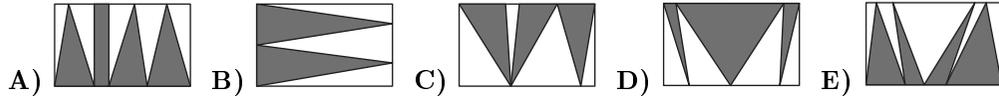

A) $\alpha = \beta$ **B)** $2\alpha + \beta = 90°$ **C)** $\alpha + \beta = 60°$ **D)** $2\beta + \alpha = 90°$
E) $\alpha + \beta = 45°$

7. How many different sums of dots can you get by rolling three standard dice simultaneously?
A) 14   **B)** 15   **C)** 16   **D)** 17   **E)** 18

8. A rectangle has been shaded in five different ways as shown. In which diagram does the shaded part have the largest area?

A)   B)   C)   D)   E)

9. The weight (in kg) of each of three kangaroos is a different whole number. The total weight of them is 97 kg. How much can the lightest of them weigh at most?
A) 1 kg   B) 30 kg   C) 31 kg   D) 32 kg   E) 33 kg

10. Five identical glasses are filled with water. Four of them contain the same amount of water. Which one contains a different amount?

A)   B)   C)   D)   E)
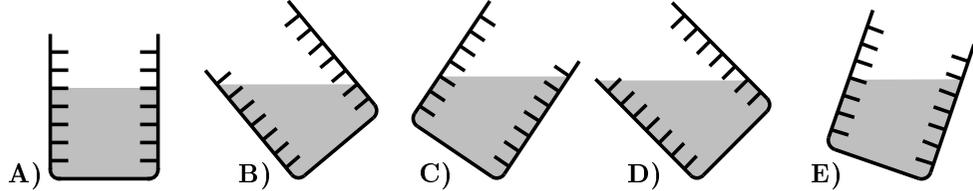
Questions for 4 points

11. On each of three pieces of paper a five-digit number is written as shown. Three of the digits are covered. The sum of the three numbers is 57263. Which are the covered digits?

A) 0, 2, 2   B) 1, 2, 9   C) 2, 4, 9   D) 2, 7, 8   E) 5, 7, 8
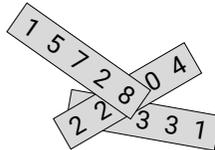
12. A square has vertices A, B, C, D labelled clockwise. Point B lies inside an equilateral triangle AEC. What is the size of angle CBE?
A) 30°   B) 45°   C) 135°   D) 145°   E) 150°

13. The numbers a, b, c, d are distinct positive integers chosen from 1 to 10. What is the least possible value $\frac{a}{b} + \frac{c}{d}$ could have?
A) $\frac{2}{10}$ B) $\frac{3}{19}$ C) $\frac{14}{45}$ D) $\frac{29}{90}$ E) $\frac{25}{72}$

14. The flag of Kanguria is a rectangle with side lengths in the ratio $5 : 3$. The flag is divided into four rectangles of equal area as shown. What is the ratio of the side lengths of the white rectangle?
A) $3 : 1$ B) $4 : 1$ C) $7 : 2$ D) $18 : 5$ E) $15 : 4$
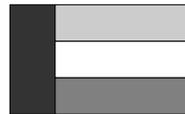
15. The triathlon consists of swimming, running, and biking. The biking is three-quarters of the total distance; the running is one-fifth; and the swimming is 2 km. What is the total distance of this triathlon, in km?
A) 10   B) 20   C) 38   D) 40   E) 60

16. The system shown consists of three pulleys with vertical sections of rope between them. The end P is moved down 24 centimetres. How many centimetres does point Q move up?
A) 24   B) 12   C) 8   D) 6   E) $\frac{24}{5}$
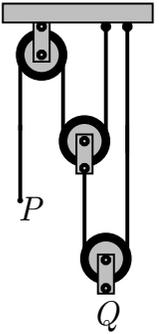
17. Some diluted juice is to be made out of concentrate and water in the ratio $1 : 7$ by volume. Juice concentrate is in a 1-litre flask, and the flask is half full. What fraction of this concentrate should be used to produce 2 litres of diluted juice?
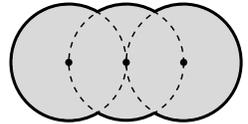A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{2}{7}$ D) $\frac{4}{7}$ E) All of the concentrate

18. The seven digits of the telephone number $\overline{aaabbbb}$ add up to the two-digit number $\overline{ab}$. What is the sum $a + b$?
A) 8   B) 9   C) 10   D) 11   E) 12

19. The given shape is made of parts of three equal circles of radius R. The middle circle has its centre at the point of tangency of the other two, as shown. What is the perimeter of the shape?
A) $\frac{10\pi R}{3}$ B) $\frac{5\pi R}{3}$ C) $\frac{2\pi R\sqrt{3}}{3}$ D) $2\pi R\sqrt{3}$ E) $4\pi R$


20. 60 apples and 60 pears are packed into boxes so that each box contains the same number of apples, and no two boxes contain the same number of pears. What is the largest possible number of boxes that can be packed in this way?
A) 20   B) 15   C) 12   D) 10   E) 6

Questions for 5 points

21. The diagram shows a net of an octahedron. When this is folded to form the octahedron, which of the labelled line segments will coincide with the line segment marked with the x?
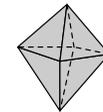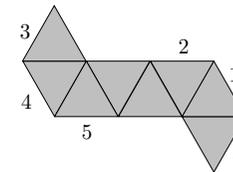
A) 1   B) 2   C) 3   D) 4   E) 5

22. Let a be the sum of all positive divisors of 1024 and b the product of all positive divisors of 1024. Then
A) $(a - 1)^5 = b$ B) $(a + 1)^5 = b$ C) $a^5 = b$ D) $a^5 - 1 = b$ E) $a^5 + 1 = b$

23. The integers from 1 to 99 are written in ascending order without gaps. The sequence of digits is then divided into triplets of digits:$(123)(456)(789)(101)(112)\ldots(979)(899)$. Which of the following is not one of the triplets?
A) (222) B) (444) C) (464) D) (646) E) (888)