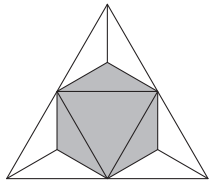
**24.** Consider a regular tetrahedron. Its four corners are cut off by four planes, each passing through the midpoints of three adjacent edges (see figure). What part of the volume of the original tetrahedron is the volume of the resulting solid?

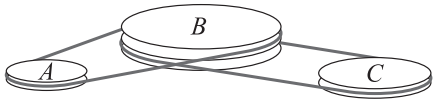


**A)** $\frac{4}{5}$ **B)** $\frac{3}{4}$ **C)** $\frac{2}{3}$ **D)** $\frac{1}{2}$ **E)** $\frac{1}{3}$

**25.** The sum of the lengths of the three sides of a right-angled triangle is equal to 18 and the sum of the squares of the lengths of the three sides is equal to 128. What is the area of the triangle?

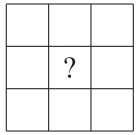
**A)** 18 **B)** 16 **C)** 12 **D)** 10 **E)** 9

**26.** A belt drive system consists of the wheels $A$, $B$ and $C$, which rotate without a slippage. $B$ turns 4 full rounds when $A$ turns 5 full rounds, and $B$ turns 6 full rounds when $C$ turns 7 full rounds. Find the perimeter of $A$ if the perimeter of $C$ is 30 cm.




**A)** 30 cm **B)** 28 cm **C)** 27 cm **D)** 24 cm **E)** 21 cm

**27.** Nine integers are written in the cells of a $3 \times 3$ table. The sum of the nine numbers is equal to 500. It is known that the numbers in any two neighboring cells (that is, cells sharing a common side) differ by 1. What is the number in the central cell?




**A)** 50 **B)** 54 **C)** 55 **D)** 56 **E)** 57

**28.** Consider the sequence $a_1, a_2, a_3, \ldots$ with $a_1 = 2017$ and $a_{n+1} = \dfrac{a_n - 1}{a_n}$, $n \geqslant 1$. Then $a_{2017} =$

**A)** $-2017$ **B)** $-\frac{1}{2016}$ **C)** $\frac{2016}{2017}$ **D)** 1 **E)** 2017

**29.** How many three-digit positive integers $\overline{abc}$ exist, such that $(a+b)^c$ is a three-digit integer and an integer power of 2?

**A)** 15 **B)** 16 **C)** 18 **D)** 20 **E)** 21

**30.** Each of the 2017 people living on an island is either a liar (and always lies) or a truth-teller (and always tells the truth). More than one thousand of them take part in a banquet, all sitting together at a round table. Each of them says: "Of the two people beside me, one is a liar and the other one a truth-teller." How many truth-tellers are there on the island at most?

**A)** 1683 **B)** 668 **C)** 670 **D)** 1344 **E)** 1343



*Lietuvos Respublikos švietimo ir mokslo ministerija*
*Kengūros konkurso organizavimo komitetas*
*Lietuvos matematikų draugija*
*VU Matematikos ir informatikos fakultetas*
*VU Matematikos ir informatikos institutas*

# KANGAROO 2017

S

**Student**
**11–12 grades**

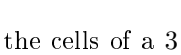
*Time allowed: 75 minutes*
*Calculators are not permitted*

## Questions for 3 points

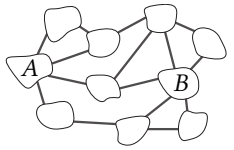
**1.** Ben likes to play with his H0-model railroad. He modeled some things in the H0-ratio 1 : 87, even a 2 cm high model of his brother. What is the real height of his brother?

**A)** 1.74 m **B)** 1.62 m **C)** 1.86 m **D)** 1.94 m **E)** 1.70 m

**2.** Peter wrote the word **KENGŪRA** on a piece of transparent glass (see figure). What will he see if he turns this piece over?


**A)**  **B)**  **C)**  **D)**  **E)** 

**3.** In the figure we see 10 islands that are connected by 15 bridges. What is the smallest number of bridges that can be eliminated in order to make it impossible to get from $A$ to $B$ by bridge?

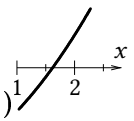


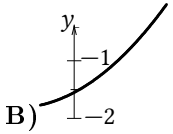

**A)** 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

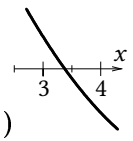
**4.** Two positive numbers $a$ and $b$ are such that 75% of $a$ equals 40% of $b$. This means that

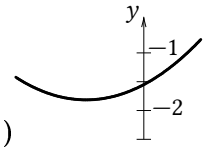
**A)** $15a = 8b$ **B)** $7a = 8b$ **C)** $3a = 2b$ **D)** $5a = 12b$ **E)** $8a = 15b$

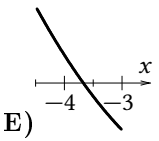
**5.** Four of the following five clippings are part of the graph of the same quadratic function. Which clipping is not part of this graph?

**A)** 

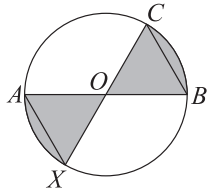

**B)** 


**C)** 


**D)** 

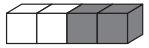

**E)** 

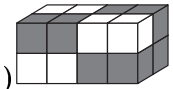
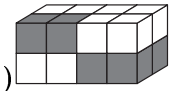
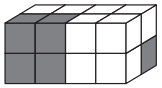
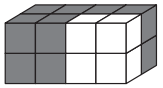
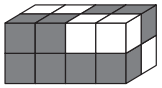

**6.** Given a circle with center $O$ and diameters $AB$ and $CX$ such that $OB = BC$. What portion of the area of the circle is shaded?




**A)** $\frac{2}{5}$ **B)** $\frac{1}{3}$ **C)** $\frac{2}{7}$ **D)** $\frac{3}{8}$ **E)** $\frac{4}{11}$

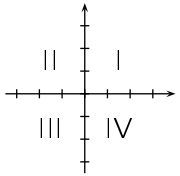
**7.** A bar consists of 2 white and 2 grey cubes glued together such that the result is a $4 \times 1 \times 1$ bar with 2 white cubes on one end and 2 grey cubes on the other end (see figure). Which figure can be built from 4 bars?



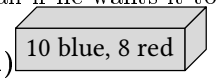
**A)**  **B)**  **C)**  **D)**  **E)** 

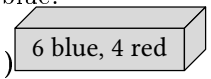
**8.** Which quadrant contains no points of the graph of the linear function $f(x) = -3.5x + 7$?

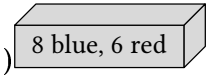
**A)** I **B)** II **C)** III **D)** IV **E)** All quadrants contain points

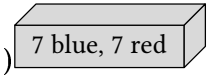


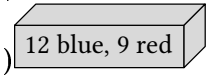

**9.** Each of the following five boxes are filled with red and blue balls as labeled. Ben wants to take one ball out of the boxes without looking. From which box should he take the ball if he wants it to be blue?

**A)** 

**B)** 

**C)** 

**D)** 

**E)** 

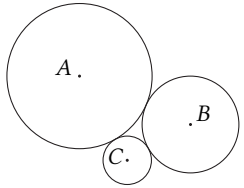

**10.** The graph of which of the following functions has the most points in common with the graph of the function $f(x) = x$?

**A)** $g_1(x) = x^2$ **B)** $g_2(x) = x^3$ **C)** $g_3(x) = x^4$ **D)** $g_4(x) = -x^4$ **E)** $g_5(x) = -x$

### Questions for 4 points

**11.** Three mutually tangent circles with centres $A$, $B$, $C$ have the radii 3, 2 and 1, respectively. What is the area of the triangle $ABC$?

**A)** 6 **B)** $4\sqrt{3}$ **C)** $3\sqrt{2}$ **D)** 9 **E)** $2\sqrt{6}$



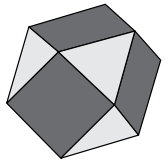

**12.** The positive number $p$ is less than 1, and the number $q$ is greater than 1. Which of the following numbers is the largest?

**A)** $p^2q$ **B)** $pq^2$ **C)** $p^2q^2$ **D)** $p^2 + q^2$ **E)** $p + q^2$

**13.** Two right cylinders $A$ and $B$ have the same volume. The radius of the base of $B$ is 10 % larger than that of $A$. How much larger is the altitude of $A$ than that of $B$?

**A)** 5 % **B)** 10 % **C)** 11 % **D)** 20 % **E)** 21 %

**14.** The faces of the polyhedron shown are either triangles or squares. Each edge is a common side of a triangle and a square. If there are 6 squares, how many triangles are there?



**A)** 5 **B)** 6 **C)** 7 **D)** 8 **E)** 9

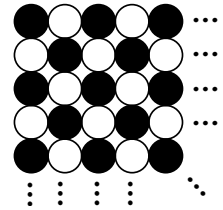
**15.** If $|x| + x + y = 5$ and $x + |y| - y = 10$ what is the value of $x + y$?

**A)** 1 **B)** 2 **C)** 3 **D)** 4 **E)** 5

**16.** The polynomial $5x^3 + ax^2 + bx + 24$ has integer coefficients $a$ and $b$. Which of the following is certainly not a root of the polynomial?

**A)** 1 **B)** $-1$ **C)** 3 **D)** 5 **E)** 6

**17.** Julia has 2017 chips: 1009 of them are black and the rest are white. She places them in a square pattern as shown, beginning with a black chip in the upper left hand corner, alternating colours in each row and each column. How many chips of each colour are left after she has completed the largest possible square?

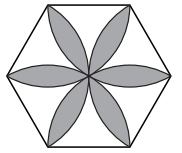


**A)** None **B)** 40 of each **C)** 40 black ones and 41 white ones **D)** 41 of each **E)** 40 white ones and 41 black ones

**18.** Two consecutive positive integers are such that the sums of the digits of each of them are multiples of 7. At least how many digits does the smaller integer have?

**A)** 3 **B)** 4 **C)** 5 **D)** 6 **E)** 7

**19.** The picture shows a regular hexagon with side lengths equal to 1. The flower was constructed with sectors of circles of radius 1 and centers in the vertices of the hexagon. What is the area of the flower?



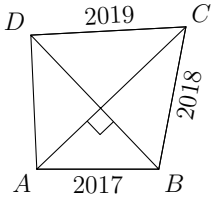
**A)** $\frac{\pi}{2}$ **B)** $\frac{2\pi}{3}$ **C)** $2\sqrt{3} - \pi$ **D)** $\frac{\pi}{2} + \sqrt{3}$ **E)** $2\pi - 3\sqrt{3}$

**20.** Tytti tries to be a good little Kangaroo, but lying is too much fun. Therefore, every third thing she says is a lie and the rest is true. (Sometimes she starts with a lie and sometimes with one or two true statements.) Tytti is thinking of a 2-digit number and is telling her friend about it: "One of its digits is a 2." "It is larger than 50." "It is an even number." "It is less than 30." "It is divisible by three." "One of its digits is a 7." What is the sum of the digits of the number Tytti is thinking of?

**A)** 9 **B)** 12 **C)** 13 **D)** 15 **E)** 17

### Questions for 5 points

**21.** In a convex quadrilateral $ABCD$ the diagonals are perpendicular. The sides have lengths $AB = 2017$, $BC = 2018$ and $CD = 2019$ (figure not to scale). What is the length of $AD$?




**A)** 2016 **B)** 2018 **C)** $\sqrt{2020^2 - 4}$ **D)** $\sqrt{2018^2 + 2}$ **E)** 2020

**22.** On the faces of a given dice these numbers appear: $-3$, $-2$, $-1$, 0, 1, 2. If you throw it twice and multiply the results, what is the probability that the product is negative?

**A)** $\frac{1}{2}$ **B)** $\frac{1}{4}$ **C)** $\frac{11}{36}$ **D)** $\frac{13}{36}$ **E)** $\frac{1}{3}$

**23.** How many positive integers have the property that the number obtained by deleting the last digit is equal to $\frac{1}{14}$ of the original number?

**A)** 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4