Questions of Kangaroo 2004

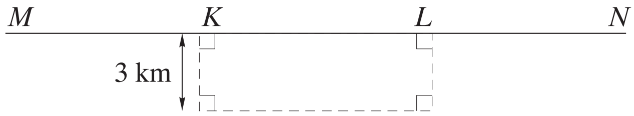
MINOR (grades 3 and 4)

3-POINT QUESTIONS

M1. How much is $2001 + 2002 + 2003 + 2004 + 2005$?
A 1015 **B** 5010 **C** 10,150 **D** 11,005 **E** 10,015

M2. Jerome was 4 years old when his sister was born. Today he blows out 9 birthday candles. What is the age difference between him and his sister?
A 4 years **B** 5 years **C** 9 years **D** 13 years **E** 14 years

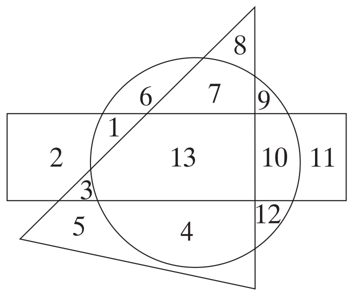
M3. In the picture below you can see a road from town M to town N (a solid line) and a detour (a dashed line) of segment KL, which is under repair. How many more kilometers does one have to travel from M to N using the detour?



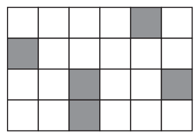
A 3 **B** 5 **C** 6 **D** 10 **E** Impossible to calculate

M4. There were some swallows on a telegraph line. All at once 5 of them flew away, and a while later 3 swallows came back. Then there were 12 swallows on the line. How many swallows were there on the line at the very beginning?
A 8 **B** 9 **C** 10 **D** 12 **E** 14

M5. Which numbers are written in the area that belongs to the rectangle and to the circle but doesn't belong to the triangle?
A 5 and 11 **B** 1 and 10 **C** 13
D 3 and 9 **E** 6, 7, and 4

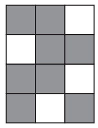


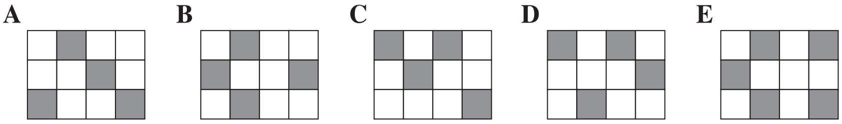
M6. How many white squares must you paint grey so that the number of grey squares is exactly half that of the white squares?
A 2 **B** 3 **C** 4 **D** 6 **E** It cannot be done



M7. Mary and Peter's classmates are standing in line. Mary has 16 students in back of her, including Peter. Peter has 14 students in front of him including Mary. Between Mary and Peter there are 7 students. How many students are there, altogether, in Mary and Peter's class?
A 37 **B** 30 **C** 23 **D** 22 **E** 16

M8. Which of the rectangles **A** to **E** can be covered by the pattern on the right-hand side in such a way that the result is a totally black rectangle?

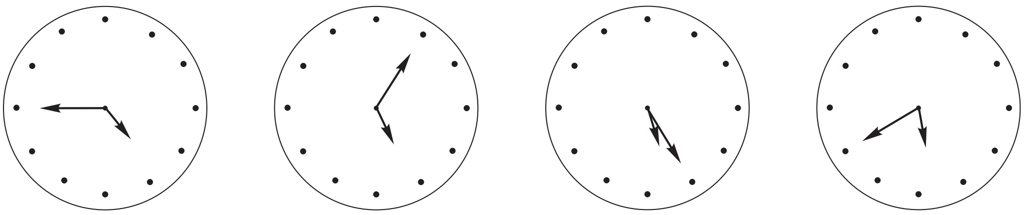




4-POINT QUESTIONS

M9. The weight of 3 apples and 2 oranges is 255 g. The weight of 2 apples and 3 oranges is 285 g. Each apple has the same weight, and each orange has the same weight. What is the weight in grams of 1 apple and 1 orange together?
A 110 **B** 108 **C** 105 **D** 104 **E** 102

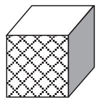
M10. In this picture there is what I saw on four different clocks at the same time. Only one of them had the right time. One was 20 minutes fast. Another 20 minutes slow. One had stopped some time ago.

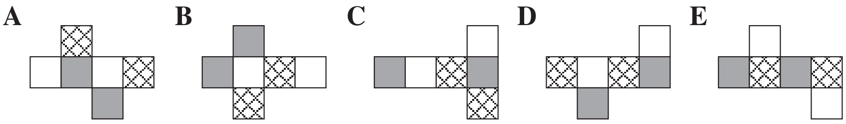


What was the right time?
A 4:45 **B** 5:05 **C** 5:25 **D** 5:40 **E** 12:00

M11. Gabriella brought Joseph a basket of apples and oranges. Joseph ate half of all the apples and one third of all the oranges. How much of the fruit could still be left in the basket?
A Half of all the fruit **B** More than half of all the fruit
C Less than half of all the fruit **D** One third of all the fruit
E Less than one third of all the fruit

M12. A cube (on the right) is colored in three colors so that each face has exactly one color and the opposite face has the same color. Which of the following developments is the development of this cube?





M13. Karen has found an old book with some missing pages. On a left-hand page the page number is 24, and the following right-hand page is numbered 45. How many leaves are missing in between?
A 9 **B** 10 **C** 11 **D** 20 **E** 21

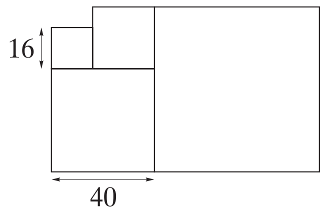
M14. Ruby is 52 days older than her classmate Irene. Last year Ruby celebrated her birthday on a Tuesday in March. On which day of the week did Irene celebrate her birthday last year?
A Monday **B** Tuesday **C** Wednesday **D** Thursday **E** Friday

M15. Which difference is not equal to $671 - 389$?
A $771 - 489$ **B** $681 - 399$ **C** $669 - 391$ **D** $1871 - 1589$ **E** $600 - 318$

M16. Inside each of the four squares of a 2×2 grid there is a number. If the sum of the numbers of the first line is 3, the sum of the numbers of the second line is 8, and the sum of the numbers of the first column is 4, what is the sum of the numbers in the second column?
A 4 **B** 6 **C** 7 **D** 8 **E** 11

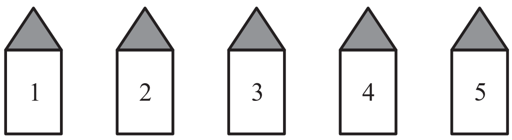
5-POINT QUESTIONS

M17. This figure is made of squares. What is the side of the biggest square?
A 24 **B** 56 **C** 64 **D** 81 **E** 100



M18. Robert has 147 euros, and Lisa has 57 euros. How many euros must Robert give to Lisa so that Robert has twice as much as Lisa?
A 11 **B** 19 **C** 30 **D** 45 **E** 49

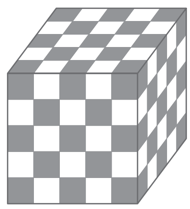
M19. There are five houses on Color Street: a blue, a red, a yellow, a pink, and a green one. The houses are numbered from 1 to 5 (see picture). The red house is the neighbor of the blue house only. The blue house stands between the green and red houses.



Which color is the house with number 3?
A Blue **B** Red **C** Yellow **D** Pink **E** Green

M20. The sum of the digits of a ten-digit number is equal to 9. What is the product of the digits of this number?
A 0 **B** 1 **C** 45 **D** $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$ **E** Depends on the given number

M21. A large cube consists of 125 small black and white cubes, such that any two adjacent faces of the small cubes have different colors, the corner cubes being black. How many small black cubes are used?
A 62 **B** 63 **C** 64 **D** 65 **E** 68



M22. One lottery ticket costs 4 euros. Three boys – Paul, Peter, and Robert – pooled their money for two tickets. Paul gave 1 euro, Peter – 3 euros, Robert – 4 euros. One of the tickets they bought won 1000 euros. The boys shared the prize fairly, i.e., according to how much money each of them had contributed. How many euros did Peter get?
A 300 **B** 375 **C** 250 **D** 750 **E** 425

M23. After three games of the soccer championship, Platypus United has scored three goals and let one past them. They get three points for a win, one point for a draw, and no points for a loss. How many points can they not have right now?
A 7 **B** 6 **C** 5 **D** 4 **E** 3

M24. This is a multiplication table. Which two letters represent the same number?
A L and M **B** P and N **C** R and S
D K and R **E** M and T

×				7
	J	K	L	56
	M	36	8	N
	P	27	6	R
6	18	S	T	42

BENJAMIN (grades 5 and 6)

3-POINT QUESTIONS

B1. How much is $1000 - 100 + 10 - 1$?
A 111 **B** 900 **C** 909 **D** 990 **E** 999

B2. Caroline wants to write the numbers 1, 2, 3, 4 in the square 4×4 in such a way that every row and every column has each number. You see how she started. What number must be put in the place of x?
A 1 **B** 2 **C** 3 **D** 4 **E** Impossible to determine

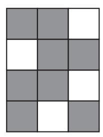
1		x	2
4	1		
	3		
	2		

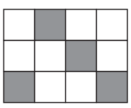
B3. The product $(10 \times 100) \times (20 \times 80)$ is equal to
A $20{,}000 \times 80{,}000$ **B** 2000×8000 **C** $2000 \times 80{,}000$ **D** $20{,}000 \times 8000$ **E** 2000×800

B4. How many hours is 360,000 seconds?
A 3 **B** 6 **C** 8.5 **D** 10 **E** More than 90

B5. If 20042003 is divided by 2004, the remainder is
A 0 **B** 1 **C** 2 **D** 3 **E** 2003

B6. Which of the rectangles **A** to **E** can be covered by the pattern on the right-hand side in such a way that the result is a totally black rectangle?



A

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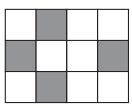
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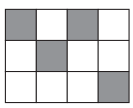
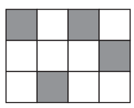
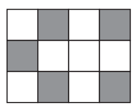

D

E


B7. Which of the following is not a factor of 2004?
A 3 **B** 4 **C** 6 **D** 8 **E** 12

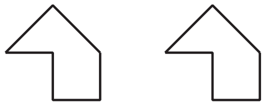
B8. The three members of a rabbit family have altogether eaten 73 carrots. The father has eaten five carrots more than the mother. The son Bunny has eaten 12 carrots. How many carrots has the mother eaten?
A 27 **B** 28 **C** 31 **D** 33 **E** 56

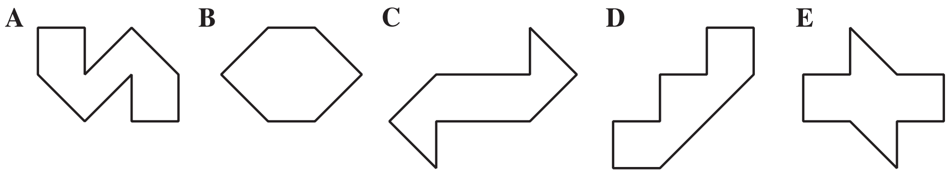
B9. Nine bus stops are equally spaced along a bus route. The distance from the first stop to the third stop is 600 m. How many meters is it from the first to the last?
A 1800 **B** 2100 **C** 2400 **D** 2700 **E** 3000

B10. The sum of the digits of a ten-digit number is equal to 9. What is the product of the digits of this number?
A 0 **B** 1 **C** 45 **D** $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$ **E** Depends of the given number

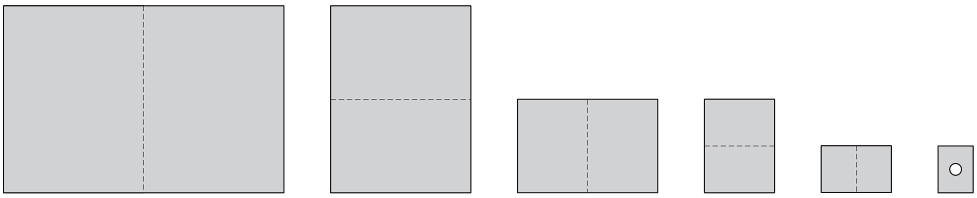
4-POINT QUESTIONS

B11. You have two identical pieces that you can turn around but not upside down. Which picture can you not make with these two pieces?



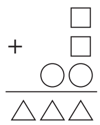


B12. Harry folds a sheet of paper five times. Then he makes a hole in the folded paper, after which he unfolds it.



How many holes has the unfolded paper?
A 6 **B** 10 **C** 16 **D** 20 **E** 32

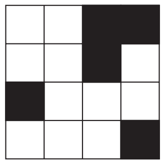
B13. Different figures represent different digits. Find the digit corresponding to the square.
A 9 **B** 8 **C** 7 **D** 6 **E** 5



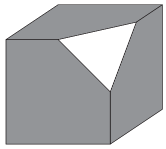
B14. The weight of 3 apples and 2 oranges is 255 g. The weight of 2 apples and 3 oranges is 285 g. Each apple has the same weight, and each orange has the same weight. What is the weight in grams of 1 apple and 1 orange together?
A 110 **B** 108 **C** 105 **D** 104 **E** 102

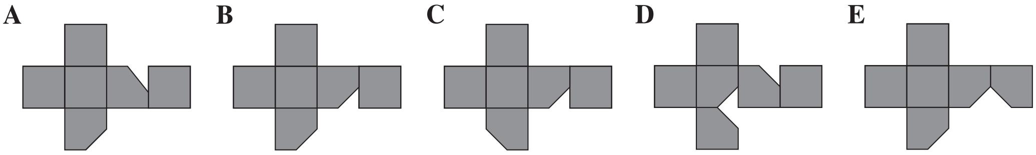
B15. The best mathematician in the 7th grade was asked to guess the positive integer about which his friends made the following statements:
Thomas: “This number is 9.”
Ronald: “This number is prime.”
Andrew: “This number is even.”
Michael: “This number is 15.”
Ronald and Thomas together made one true statement, as well as Andrew and Michael. This number is:
A 1 **B** 2 **C** 3 **D** 9 **E** 15

B16. What is the smallest number of little squares that need to be painted to get at least one axis of symmetry in the picture?
A 1 **B** 2 **C** 3 **D** 4 **E** 5

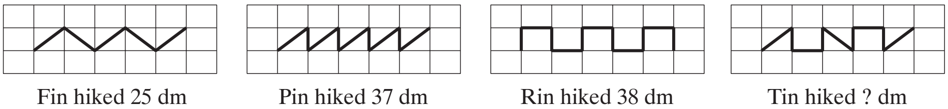


B17. We have cut off one corner of a cube. Which of the developments below is the development of the remaining part?





B18. Snail quadruplets have gone hiking on a path paved with identical rectangular tiles. The shape and length of each snail's trip is shown below.

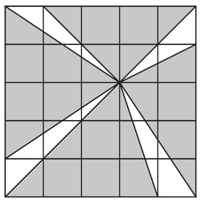


How many decimeters has the snail Tin hiked?
A 27 **B** 30 **C** 35 **D** 36 **E** 40

B19. Turtle Island has an unusual weather system: on Mondays and Wednesdays it's always rainy, on Saturdays it's foggy, and the other days are sunny. A group of tourists would like to go on a 44-day-long holiday to the island. Which day of the week should be the first day of their holiday in order to enjoy the most sunny days?
A Monday **B** Wednesday **C** Thursday **D** Friday **E** Tuesday

B20. The sum of two positive integers is equal to 77. If the first number is multiplied by 8 and the second by 6, the two products are equal. The larger of these numbers is
A 23 **B** 33 **C** 43 **D** 44 **E** 54

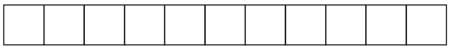
5-POINT QUESTIONS



B21. In the diagram drawn on the square grid, find the ratio of the unshaded area to the shaded area.
A $\frac{1}{4}$ **B** $\frac{1}{5}$ **C** $\frac{1}{6}$ **D** $\frac{2}{5}$ **E** $\frac{2}{7}$

B22. Ella and Emma went mushrooming. They found 70 mushrooms. $\frac{5}{9}$ of the mushrooms Ella found were boletuses, and $\frac{2}{17}$ of the mushrooms Emma has found were orange-caps. How many mushrooms did Ella find?
A 27 **B** 36 **C** 45 **D** 54 **E** 10

B23. In the picture we have 11 fields.



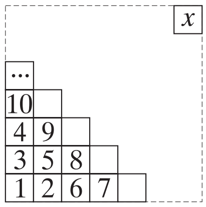
In the first field there is a 7, and in the ninth field we have a 6. What positive integer has to be written in the second field for the following condition to be valid: the sum of any three adjoining fields is equal to 21?
A 7 **B** 8 **C** 6 **D** 10 **E** 21

B24. This is a multiplication table. Which two letters represent the same number?
A L and M **B** P and N **C** R and S
D K and R **E** M and T

×				7
	J	K	L	56
	M	36	8	N
	P	27	6	R
6	18	S	T	42

B25. In a CD store two CD's have the same price. The first CD becomes 5% cheaper, and the other one increases 15% in price. Now the two prices differ by 6 euros. What is the price in euros of the cheaper CD now?
A 1.50 **B** 6 **C** 28.50 **D** 30 **E** 34.50

B26. You write a number in each square as shown in the square figure. Then, the number x cannot be:
A 128 **B** 256 **C** 81 **D** 121 **E** 400



B27. Bill divided $\underbrace{111...1}_{2004}$ by 3. The number of zeros in the quotient he obtained is equal to
A 670 **B** 669 **C** 668 **D** 667 **E** 665

B28. Imagine that you have 108 red balls and 180 green balls. You want to put all of them in bags, and there must be the same number of balls in each bag, and all the balls in each bag must be the same color. What is the minimum number of bags you need?
A 288 **B** 36 **C** 18 **D** 8 **E** 1

B29. In the Kangaroo summer camp a math competition was organized with 10 problems. Each correct answer was worth 5 points. For each incorrect answer 3 points were deducted. Everybody answered all the problems. Matt had 34 points, Zsolt had 10 points, and Gábor had 2 points. How many correct answers did they have altogether?
A 17 **B** 18 **C** 15 **D** 13 **E** 21

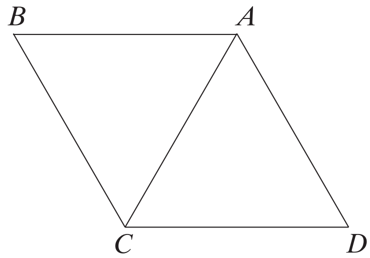
B30. A right triangle with legs of length 6 cm and 8 cm is cut out of a sheet of paper and then folded along a straight line. What can the area be, in cm^2, of the resulting polygon?
A 9 **B** 12 **C** 18 **D** 24 **E** 30

CADET (grades 7 and 8)

3-POINT QUESTIONS

C1. What is the value of $2004 - 200 \cdot 4$?
A 7216 **B** 0 **C** 1204 **D** 1200 **E** 2804

C2. An equilateral triangle ACD is rotated counterclockwise around point A. At what angle has it been rotated unen it covers equilateral triangle ABC for the first time?
A 60° **B** 120° **C** 180° **D** 240° **E** 300°



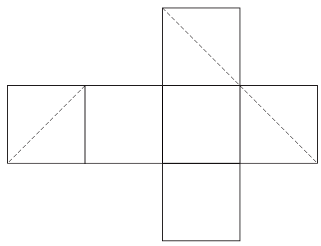
C3. We multiplied the number x by 0.5 and divided the product obtained by 3. By squaring the quotient and adding 1 we obtained 50. What is the number x equal to?
A 18 **B** 24 **C** 30 **D** 40 **E** 42

C4. Caroline wants to write the numbers 1, 2, 3, 4 in the square 4×4 in such a way that every row and every column has each of the numbers. You see how she started. How many of the 4 numbers can be written in place of x?
A 1 **B** 2 **C** 3 **D** 4 **E** Impossible to determine

1		x	
4	1		
	3		
	2		

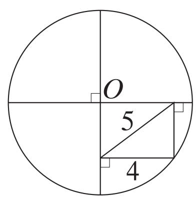
C5. The value of the expression $(1 - 2) - (3 - 4) - (5 - 6) - \cdots - (99 - 100)$ is equal to
A -50 **B** 49 **C** -48 **D** 48 **E** 50

C6. The section of a cube by a plane generates a plane figure. I have plotted this section in the development of the cube (see the picture). Can you find out what figure it is?
A An equilateral triangle
B A rectangle, but not a square
C A right triangle
D A square
E A hexagon



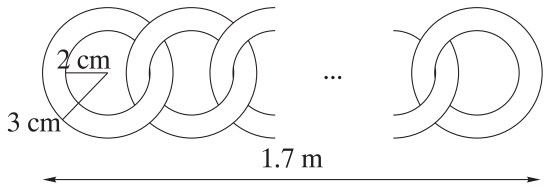
C7. We have a rectangle and decide to enlarge it by increasing both length and width by 10%. The percentage of increase in area is
A 10% **B** 20% **C** 21% **D** 100% **E** 121%

C8. The point O is the center of the circle in the picture. What is the diameter of the circle?
A 18 **B** 12 **C** 10 **D** 12.5 **E** 14



C9. An ice cream stand has five different flavors. A group of children comes to the stand, and each child buys a double scoop cone with two flavors of ice cream. If none of the children choose the same combination of flavors, and every different combination of flavors is chosen, how many children are there?
A 5 **B** 10 **C** 20 **D** 25 **E** 30

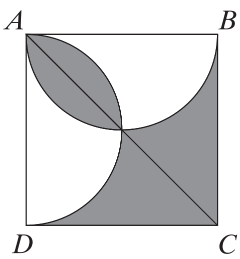
C10. We link rings together as shown in the figure below; the length of the chain is 1.7 m.



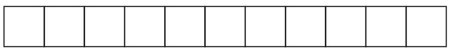
How many rings are there?
A 17 **B** 21 **C** 30 **D** 42 **E** 85

4-POINT QUESTIONS

C11. In the picture a square $ABCD$ and two semicircles with diameters AB and AD have been drawn. If $AB = 2$, what is the area of the shaded region?
A 4 **B** 8 **C** 8π **D** 2π **E** 3



C12. In the picture we have 11 fields.



In the first field there is a 7, and in the ninth field we have a 6. What positive integer has to be written in the second field for the following condition to be valid: the sum of any three adjoining fields is equal to 21?
A 7 **B** 8 **C** 6 **D** 10 **E** 21

C13. In the first year of two consecutive years there were more Thursdays than Tuesdays. Which day of the week was there more of in the second year, considering that neither of these years was a leap year?
A Tuesday **B** Wednesday **C** Friday **D** Saturday **E** Sunday

C14. ABC is an isosceles triangle with $AB = AC = 5\,\text{cm}$ and $\angle BAC > 60°$. The length of its perimeter is a whole number of centimeters. How many such triangles are possible?
A 1 **B** 2 **C** 3 **D** 4 **E** 5

C15. Romeo the ostrich is training for the Head in the Sand Competition. He put his head into the sand at 8:15 on Monday morning and having been underground for 98 hours and 56 minutes reached a new personal record. When did Romeo pull his head out of the sand?
A On Thursday at 5:19 **B** On Thursday at 5:41 **C** On Thursday at 11:11
D On Friday at 5:19 **E** On Friday at 11:11

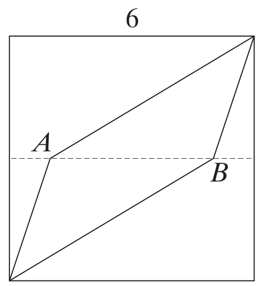
C16. Somebody has a large amount of building bricks $1 \times 2 \times 3$. What is the smallest number of bricks needed to build a cube?
A 12 **B** 18 **C** 24 **D** 36 **E** 60

C17. Each of five children thinks of a number, which can be either 1, 2, or 4. Their numbers are multiplied. Which number could be the result?
A 100 **B** 120 **C** 256 **D** 768 **E** 2048

C18. The average age of grandmother, grandfather, and 7 grandchildren is 28 years. The average age of 7 grandchildren is 15 years. Find the age of grandfather, if it is known that grandfather is 3 years older than grandmother.
A 71 **B** 72 **C** 73 **D** 74 **E** 75

C19. There were more than two kangaroos in the enclosure. One kangaroo said, “There are 6 of us here,” and jumped out of the enclosure. During each consecutive minute one kangaroo jumped out of the enclosure and said, “Everybody who jumped out before me was lying.” It continued until there were no kangaroos in the enclosure. How many kangaroos told the truth?
A 0 **B** 1 **C** 2 **D** 3 **E** 4

C20. In a square with sides of length 6 the points A and B are on a line joining the midpoints of the opposite sides of the square (see the figure). When you draw lines from A and B to two opposite vertices, you divide the square in three parts of equal area. What is the length of AB?
A 3.6 **B** 3.8 **C** 4 **D** 4.2 **E** 4.4

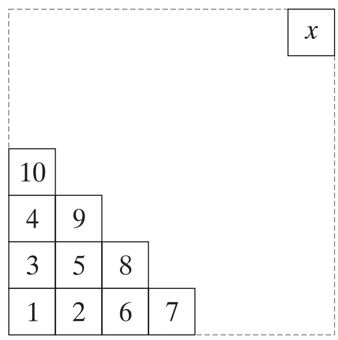


5-POINT QUESTIONS

C21. A woman goes from a city to the beach at 30 km/h. On the return trip her speed is 10 km/h. What is her average speed for the whole trip?
A 12 **B** 15 **C** 20 **D** 22 **E** 25

C22. John decided to put some of his magazines on his bookshelf. They have either 48 or 52 pages. Which one of these numbers cannot be the total number of pages of the magazines on the bookshelf?
A 500 **B** 524 **C** 568 **D** 588 **E** 620

C23. You write a number in each square as shown in the square figure. Then, the number x cannot be:
A 128 **B** 256 **C** 81 **D** 121 **E** 400



C24. If a and b are positive integers, neither of which is divisible by 10, and if $ab = 10{,}000$, then the sum $a + b$ equals
A 1024 **B** 641 **C** 1258 **D** 2401 **E** 1000

C25. After one operation, the triplet (a, b, c) turns into triplet $(b + c,\ c + a,\ a + b)$. After 2004 successive operations, the triplet $(1, 3, 5)$ turned into a triplet (x, y, z). What is the difference $x - y$ equal to?
A -2 **B** 2 **C** 4008 **D** 2004 **E** $(-2)^{2004}$

C26. This is a multiplication table. Which two letters represent the same number?
A L and M **B** P and N **C** R and S
D K and R **E** M and T

×				7
	J	K	L	56
	M	36	8	N
	P	27	6	R
6	18	S	T	42

C27. Some positive integers are written on the faces of a cube, and at each vertex we write the number equal to the product of the numbers on the three faces adjacent to that vertex. The sum of the numbers at the vertices is 70. Then the sum of the numbers on the faces is:
A 12 **B** 35 **C** 14 **D** 10 **E** Impossible to determine

C28. The number 2004 is divisible by 12, and the sum of its digits is equal to 6. Altogether, how many four-digit numbers have these two properties?
A 10 **B** 12 **C** 13 **D** 15 **E** 18

C29. A right triangle with legs of length 6 cm and 8 cm is cut out of a sheet of paper and then folded along a straight line. What can the area be, in cm^2, of the resulting polygon?
A 9 **B** 12 **C** 18 **D** 24 **E** 30

C30. In the Kangaroo summer camp a math competition was organized with 10 problems. Each correct answer was worth 5 points. For each incorrect answer 3 points were deducted. Everybody answered all the problems. Matt had 34 points, Zsolt had 10 points, and Gábor had 2 points. How many correct answers did they have altogether?
A 17 **B** 18 **C** 15 **D** 13 **E** 21

JUNIOR (grades 9 and 10)

3-POINT QUESTIONS

J1. The value of the expression $(1-2)-(3-4)-(5-6)-\cdots-(99-100)$ is equal to
A -50 **B** 49 **C** -48 **D** 48 **E** 50

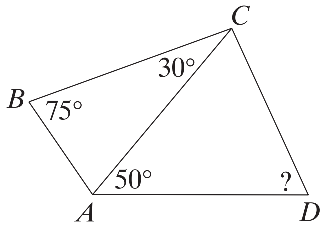
J2. Edward has 2004 marbles. Half of them are blue, one quarter are red, and one sixth are green. How many marbles are of some other color?
A 167 **B** 334 **C** 501 **D** 1001 **E** 1837

J3. A pyramid has 7 faces. How many edges does it have?
A 7 **B** 9 **C** 12 **D** 14 **E** 21

J4. The ground plan of a building has a rectangular shape with parameters of $40\,\text{m} \times 60\,\text{m}$. In the diagram the ground plan of the building has a perimeter of 100 cm. What is the scale of the diagram?
A 1:100 **B** 1:150 **C** 1:160 **D** 1:170 **E** 1:200

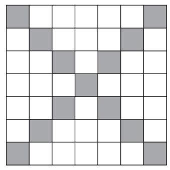
J5. Tom and Ron both had some one-euro coins. When Tom got 5 more coins from his grandfather, he had twice as many coins as Ron. And if Tom now gave 12 coins to his grandmother, he would have half as many coins as Ron. How many coins did Tom have at the very beginning?
A 5 **B** 7 **C** 9 **D** 11 **E** 45

J6. Some angles in the quadrilateral $ABCD$ are shown in the figure. If $BC = AD$, then what is the angle ADC?
A 30° **B** 50° **C** 55° **D** 65° **E** 70°

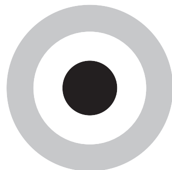


J7. There are some boletuses and orange-caps in a basket – 30 mushrooms altogether. If we randomly take out 12 mushrooms, there will be at least one orange-cap among them. If we randomly take out 20 mushrooms, there will be at least one boletus among them. How many boletuses are there in the basket?
A 11 **B** 12 **C** 19 **D** 20 **E** 21

J8. In a square 2003×2003, the squares 1×1 on the diagonals are colored (like in the picture, where the square is 7×7). How many white squares are there?
A 2002^2 **B** 2002×2001 **C** 2001^2 **D** 2003×2002
E $2003^2 - 2004$



J9. The dartboard shown consists of an inner black circle and 2 rings around it. The width of each ring is equal to the radius of the black circle. How many times greater is the area of the grey ring than the area of the inner black circle?
A 2 **B** 3 **C** 4 **D** 5 **E** 6

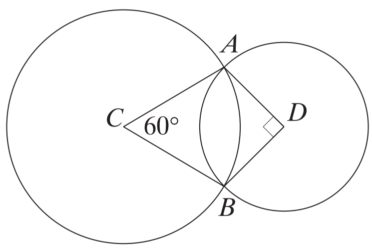


J10. After gathering 770 nuts, three girls divided them in proportion to their ages. For every 3 nuts Oxana took, Ira took 4. For every 7 nuts Natalya took, Ira took 6. How many nuts did the youngest girl get?
A 264 **B** 256 **C** 218 **D** 198 **E** 180

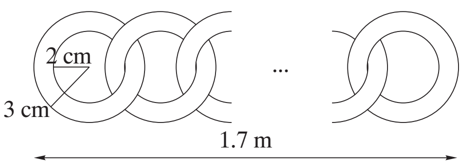
4-POINT QUESTIONS

J11. Each of five children thinks of a number, which can be either 1, 2, or 4. Their numbers are multiplied. Which number could be the result?
A 100 **B** 120 **C** 256 **D** 768 **E** 2048

J12. The circles with centers C and D meet at the points A and B, as shown. Angle $ACB = 60°$ and angle $ADB = 90°$. How many times longer is the radius of the larger circle than the radius of the smaller circle?
A $\frac{4}{3}$ **B** $\sqrt{2}$ **C** $\frac{3}{2}$ **D** $\sqrt{3}$ **E** 2

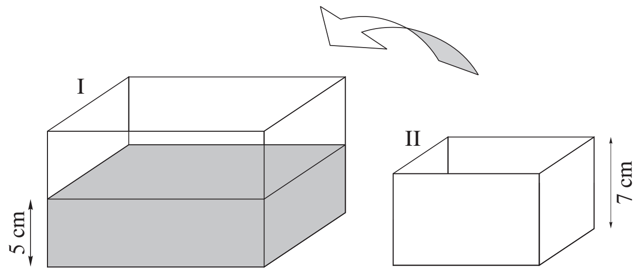


J13. We link rings together as shown in the figure below; the length of the chain is 1.7 m.



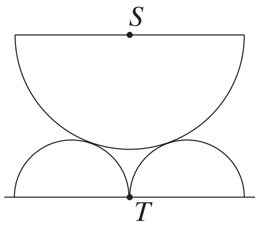
How many rings are there?
A 17 **B** 21 **C** 30 **D** 42 **E** 85

J14. In tank I, whose base has an area of $2\,\text{dm}^2$ and whose height is $10\,\text{cm}$, the water is 5 cm high. An empty tank II with a base of area $1\,\text{dm}^2$ and a height of 7 cm is placed in tank I. The water of tank I rises, of course, and spills over into tank II.
What level does the water reach in tank II?
A 1 cm **B** 2 cm **C** 3 cm
D 4 cm **E** 5 cm



J15. The hour hand of a clock is 4 cm long, and the minute hand is 8 cm long. What is the ratio of the distances travelled by the tips of the two hands between 2 pm and 5 pm?
A 1:2 **B** 1:4 **C** 1:6 **D** 1:12 **E** 1:24

J16. Three semi-circles, the diameters of two of which are equal to 4 and of the third to 8, are arranged as seen in the picture. What is the distance from the center S of the bigger semi-circle to the tangent point T of the smaller semi-circles?
A 6 **B** $\sqrt{32}$ **C** 5.7 **D** $\sqrt{40}$ **E** 5



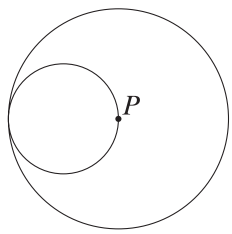
J17. A quiz has twenty questions with seven points awarded for each correct answer, two points deducted for each wrong answer, and zero for each question omitted. Andrew scores 87 points. How many questions did he omit?
A 2 **B** 3 **C** 4 **D** 5 **E** 6

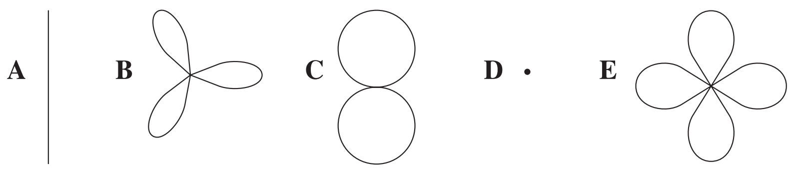
J18. Caroline wants to write the numbers 1, 2, 3, 4 in the square 4×4 in such a way that every row and every column has each of the numbers. You see how she started. In how many different ways can she finish?
A 1 **B** 2 **C** 4 **D** 16 **E** 128

1			
2	1		
	3		
	4		

J19. How many numbers exist between 100 and 200 which can have only the prime factors 2 and 3?
A 1 **B** 3 **C** 4 **D** 5 **E** 6

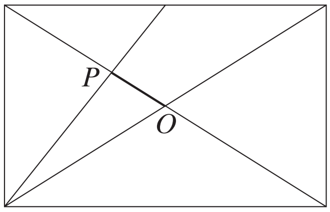
J20. The diagram shows two tangential circles with radii in the ratio 1:2. The smaller circle rolls around the inside of the large circle. Which of the following is the path traced out by the point P of the smaller circle?





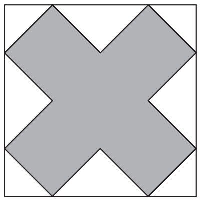
5-POINT QUESTIONS

J21. In a rectangle we draw both diagonals and the segment which joins a vertex with the midpoint of one of the sides, as shown in the picture. What is the result of dividing the length of the diagonal by the length of segment OP?
A 3 **B** 6 **C** $\frac{13}{3}$ **D** 4
E It depends on the dimensions of the rectangle



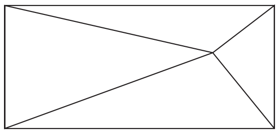
J22. The real numbers a and b have different signs. Which of the numbers given below is the largest one?
A $|a^2 - b^2|$ **B** $\left(|a| - |b|\right)^2$ **C** $(a - b)^2$ **D** $(a + b)^2$ **E** $a^2 + b^2$

J23. The diagram shows a square and an equilateral right-angled cross-shaped dodecagon. The length of the perimeter of the dodecagon is 36 cm. What, in cm^2, is the area of the square?
A 48 **B** 72 **C** 108 **D** $36\sqrt{2}$ **E** 144



J24. How many 3-digit numbers smaller than 200 have the property that the number $(n + 1)(n + 2)(n + 3)$ is divisible by 7?
A 42 **B** 38 **C** 34 **D** 28 **E** 16

J25. A rectangle is divided into 4 triangles as shown in the figure. Of the following possibilities for the areas of the triangles at most one can be true. Which one is it?
A 4, 5, 8, 9 **B** 3, 5, 6, 7 **C** 5, 6, 7, 12
D 10, 11, 12, 19 **E** 5, 6, 8, 10



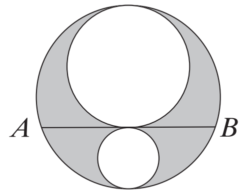
J26. This is a multiplication table. Which two letters represent the same number?
A L and M **B** P and N **C** R and S
D K and R **E** M and T

×				7
	J	K	L	56
	M	36	8	N
	P	27	6	R
6	18	S	T	42

J27. After one operation, the triplet (a, b, c) turns into triplet $(b + c,\ c + a,\ a + b)$. After 2004 successive operations, the triplet $(1, 3, 5)$ turned into a triplet (x, y, z). What is the difference $x - y$ equal to?
A -2 **B** 2 **C** 4008 **D** 2004 **E** $(-2)^{2004}$

J28. How many 8-digit numbers $\overline{a_1a_2a_3a_4a_5a_6a_7a_8}$ whose digits can only be 0s or 1s ($a_1 = 1$) have the property $a_1 + a_3 + a_5 + a_7 = a_2 + a_4 + a_6 + a_8$?
A 2^7 **B** 35 **C** 49 **D** 16 **E** 32

J29. The area of the shaded shape is equal to 2π (see the picture). What is the value of the chord AB?
A 1 **B** 2 **C** 3 **D** 4 **E** It's impossible to determine.



J30. All the integers from 1 to 10,000 were written down on a blackboard. After that the numbers that are not divisible by 5 or 11 were erased. Then the 2004th element of the sequence obtained was:
A 1000 **B** 5000 **C** 10,000 **D** 6545 **E** 7348

STUDENT (grades 11 and 12)

3-POINT QUESTIONS

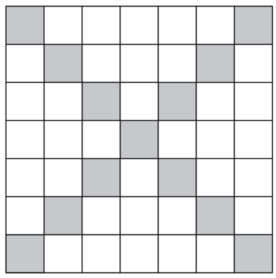
S1. If m pens are bought at n euros each, and n pens at m euros each ($m \neq n$), then the average cost per pen, in euros, is:
A 1 **B** $\frac{m+n}{2}$ **C** $\frac{2mn}{m+n}$ **D** mn **E** $\sqrt{mn}$

S2. A pyramid has 17 faces. How many vertices does it have?
A 16 **B** 17 **C** 18 **D** 32 **E** 34

S3. The smallest real number satisfying the inequality $x^2 - 2004 \leqslant 0$ is:
A -2004 **B** 2004 **C** 0 **D** $\sqrt{2004}$ **E** $-\sqrt{2004}$

S4. Each Martian has one, two, or three tentacles on its head. Exactly 1% of the Martian population consists of individuals with three tentacles, exactly 97% comprise Martians with two tentacles, and the remaining 2% consists of individuals with one tentacle. What percent of Martians have more tentacles on their head than the average of the whole Martian population?
A 1% **B** 3% **C** 97% **D** 98% **E** 99%

S5. In a square of side s, where s is an odd integer, the squares of side 1 on the diagonals are colored (like in the picture, where the square is of side 7). How many white squares are there?
A s^2+1-2s **B** s^2+4-4s **C** $2s^2+1-4s$ **D** s^2-1-2s
E s^2-2s



S6. How many two-digit numbers exist whose square and cube end in the same digit?
A 1 **B** 9 **C** 10 **D** 21 **E** More than 30

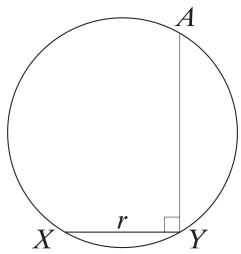
S7. Square $ABCD$ consists of 18 smaller squares, 17 of which have sides equal to 1. The area of the square $ABCD$ is:
A 25 **B** 49 **C** 81 **D** 100 **E** 225

S8. How many right triangles can be formed by joining three vertices of a given regular 14-gon?
A 72 **B** 82 **C** 84 **D** 88 **E** Other answer

S9. This is a multiplication table. What two letters represent the same number?
A L and M **B** P and N **C** R and S **D** K and R **E** M and T

×				7
	J	K	L	56
	M	36	8	N
	P	27	6	R
6	18	S	T	42

S10. On the circumference of radius r three points X, Y and A are marked such that $XY = r$, $XY \perp AY$ (see the figure).
How many degrees has the angle XAY?
A $22\frac{1}{2}$ **B** 30 **C** 45 **D** 60 **E** $67\frac{1}{2}$

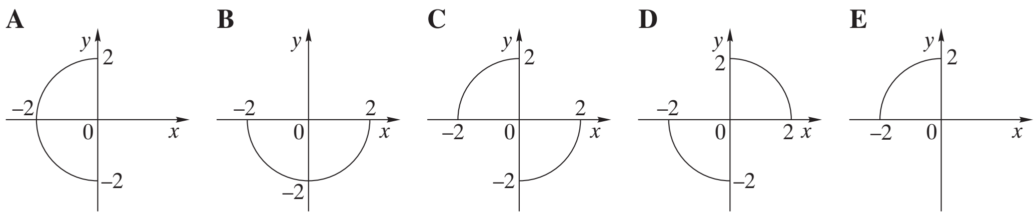


4-POINT QUESTIONS

S11. In the plane Oxy, how many squares with vertex $A(-1; -1)$ exist such that at least one of the coordinate axes is an axis of symmetry of the square?
A 2 **B** 3 **C** 4 **D** 5 **E** 6

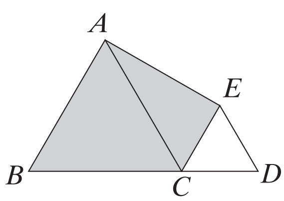
S12. There are 100 cards in a non-transparent envelope, numbered with integers from 1 to 100. There is a different number on each card. What is the smallest number of cards we have to pull out of the envelope at random to be sure that the product of the numbers on the chosen cards is divisible by 4?
A 4 **B** 52 **C** 50 **D** 48 **E** 96

S13. The set of all pairs (x, y) which satisfy conditions $xy \leqslant 0$ and $x^2 + y^2 = 4$ is on the graph:



S14. In the figure the two equilateral triangles ABC and ECD have sides of length 2 and 1 respectively. The area of the quadrilateral $ABCE$ is:

A $\frac{5\sqrt{3}}{3}$ **B** $\frac{4+5\sqrt{3}}{5}$ **C** 3 **D** $\frac{6+\sqrt{3}}{4}$ **E** $\frac{3\sqrt{3}}{2}$

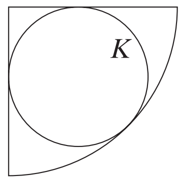


S15. How many positive integers can be written as $a_0 + a_1 \cdot 3 + a_2 \cdot 3^2 + a_3 \cdot 3^3 + a_4 \cdot 3^4$ if a_0, a_1, a_2, a_3, a_4 belong to the set $\{-1, 0, 1\}$?
A 5 **B** 80 **C** 81 **D** 121 **E** 243

S16. The number $\left(\sqrt{22 + 12\sqrt{2}} - \sqrt{22 - 12\sqrt{2}}\right)^2$ is
A negative **B** equal to zero **C** a fourth power of a non-zero integer
D equal to $11\sqrt{2}$ **E** a positive integer divisible by 5

S17. How many vertices are there in a regular polygon the sum of whose interior angles is one seventh of that of a regular 16-gon?
A 3 **B** 4 **C** 6 **D** 7 **E** 10

S18. A circle K is inscribed in a quarter circle with radius 6 as shown in the figure. What is the radius of circle K?
A $\frac{6-\sqrt{2}}{2}$ **B** $\frac{3\sqrt{2}}{2}$ **C** 2.5 **D** 3 **E** $6(\sqrt{2} - 1)$



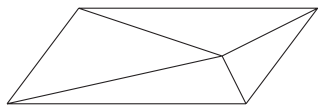
S19. For a geometric sequence (a_n) the following inequalities hold: $a_3 < a_2 < a_4$. Then
A $a_3a_4 > 0$ **B** $a_2a_3 < 0$ **C** $a_2a_4 < 0$ **D** $a_2 < 0$ **E** $a_2a_3 > 0$

S20. What is the second digit from the right of the number 11^{2004}?
A 0 **B** 1 **C** 2 **D** 3 **E** 4

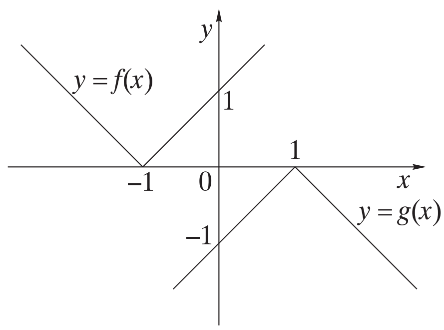
5-POINT QUESTIONS

S21. An election was held in Herbville. Every voter who voted for the Broccoli Party had already eaten broccoli. Of the remaining voters who voted for other parties 90% had never eaten broccoli. What percent did the Broccoli Party get in the election if precisely 46% of all voters in the election had eaten broccoli?
A 40% **B** 41% **C** 43% **D** 45% **E** 46%

S22. A parallelogram is divided into 4 triangles as shown in the figure. Of the following possibilities for the areas of the triangles at most one can be true. Which one is it?
A 4, 5, 8, 9 **B** 3, 5, 6, 7 **C** 5, 6, 7, 12
D 10, 11, 12, 19 **E** 5, 6, 8, 10



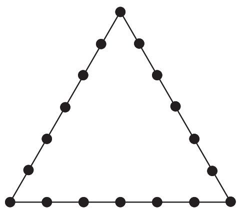
S23. The figure shows graphs of functions f and g defined on real numbers. Each graph consists of two perpendicular halflines. Which equality is satisfied for every real number x?
A $f(x) = -g(x) + 2$
B $f(x) = -g(x) - 2$
C $f(x) = -g(x + 2)$
D $f(x + 2) = -g(x)$
E $f(x + 1) = -g(x - 1)$



S24. An equilateral triangle ABC with sides of length 4 is given. The radius of the circular arc, with center at A, which divides the triangle into two parts of equal area is:
A $\sqrt{\frac{12\sqrt{3}}{\pi}}$ **B** $\sqrt{\frac{24\sqrt{3}}{\pi}}$ **C** $\sqrt{\frac{30\sqrt{3}}{\pi}}$ **D** $\frac{6\sqrt{3}}{\pi}$ **E** $\sqrt{\frac{48\sqrt{3}}{\pi}}$

S25. A game starts with a sequence of two hundred zeroes. In the first round we add 1 to every number. In the second round we add 1 to the second number and to every second number after it. In the third round we add 1 to the third number and to every third number after it, and so on. What number is in the 120th position after two hundred rounds?
A 16 **B** 12 **C** 20 **D** 24 **E** 32

S26. How many triangles can be drawn with vertices in the 18 points shown in the figure?



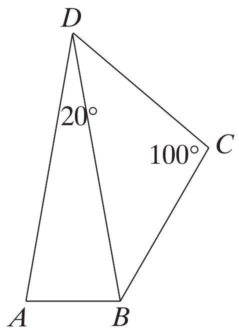
A 816 **B** 711 **C** 777 **D** 717 **E** 811

S27. If the sum of all the numbers that can be formed by permutation of the three different digits $0 < a < b < c$ is 1554, what is the value of c?
A 3 **B** 4 **C** 5 **D** 6 **E** 7

S28. The number $m = 999\ldots9$ consists of 999 nines. What is the sum of the digits of m^2 ?
A 8982 **B** 8991 **C** 9000 **D** 9009 **E** 9018

S29. $\sin^8 75^\circ - \cos^8 75^\circ$ is equal to:
A $\frac{\sqrt{3}}{2}$ **B** $\sqrt{3}$ **C** $\frac{7\sqrt{3}}{16}$ **D** 1 **E** 0

S30. Let $ABCD$ be a convex quadrilateral with an area of 1 where AB and BD are the bases of two isosceles triangles ADB and BCD respectively (as shown).



The product $AC \cdot BD$ is equal to:
A $\frac{\sqrt{3}}{3}$ **B** $\frac{2\sqrt{3}}{3}$ **C** $\sqrt{3}$ **D** $\frac{4\sqrt{3}}{3}$ **E** other answer